CPM Schedule Summarizing Function of the Beeline Diagramming Method

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Abstract
The schedule hierarchy in a construction project generally has multiple levels. The highest level is a milestone schedule, and is represented in bar chart format. The middle level is an integrated project schedule, and is represented in the Critical Path Method (CPM) format. The lowest level is a detailed working schedule, and is usually represented by a bar chart. Traditional scheduling techniques, such as the Arrow Diagramming Method, or the Precedence Diagramming Method, cannot represent all the schedules within a schedule hierarchy in identical formats. However, the Beeline Diagramming Method (BDM) technique can represent all kinds of schedule, within a schedule hierarchy, in CPM format. I describe the basic concepts, principles, interpretation methods, and schedule computation methods of the BDM as a new networking technique that can represent all kinds of overlapping relationships between activities. I then present an example of representing all levels of schedules within a schedule hierarchy, using the BDM technique, as well as BDM networks of levels 2, 3, and 4, in a real construction project.

Keywords: CPM; Arrow Diagramming Method; Precedence Diagramming Method; Beeline Diagramming Method; Schedule Summarizing

1. Introduction
Network schedules have contributed significantly to the planning, control, and on-time completion of construction projects (Callahan 1992). The Critical Path Method (CPM) has gradually increased in importance in the construction industry over the last several decades. It integrates overall project management functions, such as scheduling, cost control, and resource planning.

By the middle of the 1980s, the CPM was widely applied in the construction industry, based on the Arrow Diagramming Method (ADM), first introduced by Du Pont in 1956. The ADM has since been replaced by the Precedence Diagramming Method (PDM), proposed by Fondahl in 1961. In the PDM, overlapping relationships between activities are represented by four combinations, that connect the starting and finishing points of two consecutive activities (Ponce-Campos 1972). Confining overlapping relationships to the starting and finishing points is limiting, however, since the overlapping could happen at any point during the activity's duration. If overlapping relationships occur at any middle point, the PDM cannot properly show the relationships.

A scheduling technique must represent all kinds of relationships between activities realistically, and efficiently. I therefore describe the basic concept, principle, interpretation methods, and schedule computation methods of the Beeline Diagramming Method (BDM), a new networking technique that can represent overlapping relationships at any point of the activities, and not limit the relationships to the starting and finishing points (Kim 2010). This new networking technique will allow project teams to efficiently create more realistic project schedule plans, by representing all kinds of relationships between activities with improved flexibility.

In the schedule hierarchy of a construction project, the highest level is a milestone schedule, and is represented in bar chart format. The middle level is an integrated project schedule, and is represented in CPM format. The lowest level is a detailed working schedule, and is usually represented by a bar chart. Traditional scheduling techniques, such as the Arrow Diagramming Method (ADM), or the Precedence Diagramming Method (PDM), cannot represent all the schedules within a schedule hierarchy in identical formats. However, the BDM can represent all the schedules within the schedule hierarchy in CPM format. This paper presents the CPM schedule summarizing function in the BDM network, then presents BDM networks of levels 2, 3, and 4 in a real construction project that has an identical CPM format.

2. Research Organization
The rest of this paper is organized as follows. First, I describe the basic concepts and principles, the organization and interpretation of relationships, and the schedule computation methods of the Beeline Diagramming Method (BDM), a new networking technique that can represent overlapping relationships at any point of the activities, and not limit the relationships to the starting and finishing points (Kim 2010). This new networking technique will allow project teams to efficiently create more realistic project schedule plans, by representing all kinds of relationships between activities with improved flexibility.

In the schedule hierarchy of a construction project, the highest level is a milestone schedule, and is represented in bar chart format. The middle level is an integrated project schedule, and is represented in CPM format. The lowest level is a detailed working schedule, and is usually represented by a bar chart. Traditional scheduling techniques, such as the Arrow Diagramming Method (ADM), or the Precedence Diagramming Method (PDM), cannot represent all the schedules within a schedule hierarchy in identical formats. However, the BDM can represent all the schedules within the schedule hierarchy in CPM format. This paper presents the CPM schedule summarizing function in the BDM network, then presents BDM networks of levels 2, 3, and 4 in a real construction project that has an identical CPM format.

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3. Beeline Diagramming Method

3.1 Basic Concept, Principle, and Characteristics

The Beeline Diagramming Method (BDM) was proposed as a new networking technique by the author (2010). The basic concept of the BDM is to represent the overlapping relationship of two consecutive activities by the shortest straight line; this uses an arrow to represent the direction of workflow. The BDM connects any point of the predecessor to any point of the successor. This research defines the shortest straight line, which indicates a very direct or quick path or trip, as the "beeline" (Wiktionary 2009). Fig.1. shows the basic concept of the BDM; a beeline connects the middle point of the preceding activity A to the middle point of the succeeding activity B. The BDM has only one principle: The BDM represents the single or multiple overlapping relationships of two consecutive activities in the network by a beeline or beelines in any circumstance. Building on the basic concept and principle of the BDM, its characteristics are as follows.

First, the BDM simplifies the overlapping relationships into one beeline. Thus, the complicated process of the Precedence Diagramming Method (PDM), which includes the positioning of activities, the selection of linkage types, and the calculation of the lead-time for the selected linkage, is eliminated.

Second, the BDM permits multiple overlapping relationships by means of multiple beelines between two consecutive activities. It therefore overcomes the limitations of the compound relationships found in the PDM, which has only two overlapping linkages.

3.2 Linkage Representation Types in the BDM

Linkage relationships between two consecutive activities in the BDM are represented differently from those in the PDM. Linkage relationships in the BDM can be represented at any middle point between two consecutive activities; the PDM, in contrast, represents linkage relationships only by FS, SS, FF, and SF relationships with lead-time between the starting and finishing points.

This research proposes three types of linkage representations in the BDM. The first is the "N-N" type shown in Fig.2. This type represents two consecutive activities that are mutually connected at any point in days after their respective starts.

The initial "N" in Fig.2. refers to the days that have elapsed from the start date of the preceding activity; the latter "N" refers to the days that have elapsed from the start date of the succeeding activity; the "-" is the separation indicator between the two Ns. An example of the first representation type is illustrated in Fig.3. In this Figure, two consecutive activities that have inseparable work days are connected by a "7-4" type—between a point of 7 days after the start date of the preceding activity A and a point of 4 days after the start date of the succeeding activity B.

Fig.3. An Example by the Elapsed Days

The second type is "<N>", shown in Fig.4., wherein the successor starts some days after the completion of the predecessor.

Fig.4. Linkage Representation by Second Type

The "N" in Fig.4. refers to the lead-time to be passed after the completion of the preceding activity. The initial "<" and latter ">" indicate the lead-time space indicators. An example of the second representation type is illustrated in Fig.5.

Fig.5. An Example by Second Type

Two consecutive activities in the BDM are connected by "<4>", wherein the succeeding activity B starts after the preceding activity A has been completed for 4 days.

The third type represents the multiple linkage relationships between two consecutive activities by the elapsed days or the second linkage type. Schedule computations will continue to be performed independently for each individual linkage. Fig.6. shows

Fig.6. An Example of the Multiple Beeline Relationships
an example of the multiple beeline relationships between two activities that have multiple milestones.

3.3 Schedule Computation of the BDM

3.3.1 Forward Pass Computation

Forward pass computation determines both the early start date (ESD) and the early finish date (EFD) for the activities in the BDM network.

\[ \text{ESD}_J = \max_{i \in I} \text{ESD}_i + d_i - d_J \] (1)
\[ \text{EFD}_J = \text{ESD}_J \] (2)

The symbol \( \max \) in equation (1) means that the maximization is to be over all the beelines IJ that are merged into activity J. This research verifies equation (1) through the simple example of the multiple versus single BDM relationship. Fig.8. shows the multiple versus single relationship of the BDM wherein activities A, B, and C are merged into activity D. Thus activities A and D have a "7-3" BDM relationship, activities B and D have a "7-1" BDM relationship, and activities C and D have a "8-6" BDM relationship.

The ESD\(_D\) of the succeeding activity D in the BDM relationships with the preceding activities A, B, and C is calculated by applying equation (1) as follows: the first ESD\(_D\) of the succeeding activity D from the "7-3" relationship with activity A is calculated as ESD\(_D\) = 10 + 7 - 3 = 14; the second ESD\(_D\) from the "7-1" relationship with activity B is determined as ESD\(_D\) = 5 + 7 - 1 = 11; and the third ESD\(_D\) from the "8-6" relationship with activity C is computed as ESD\(_D\) = 13 + 8 - 6 = 15. The maximum value of "15" then is selected as the ESD\(_D\) of the succeeding activity D and the EFD\(_D\) is calculated as EFD\(_D\) = 15 + 12 = 27 by applying equation (2).

From the above, the forward pass computation of the BDM relationship proposed in this research is proved to be simple, obvious, and reasonable.

3.3.2 Backward Pass Computation

Backward pass computation determines the late start date (LSD) and the late finish date (LFD) of the activities in the BDM network. Backward pass computations in the CPM network calculate the LFD of the preceding activity first, and then determine the LSD by subtracting the duration of the preceding activity from the LFD. Due to the characteristics of the BDM network, the LSD of an activity is calculated first and the LFD is computed by adding its duration to the LSD.

Fig.9. illustrates the single versus multiple relationship of the BDM wherein activity I bursts into...
activities J1, J2, and J3.Activities I and J1 have "d_{i},d_{j}" of the BDM relationship, activities I and J2 have "d_{i},d_{j}" of the BDM relationship, and activities I and J3 have "d_{i},d_{j}" of the BDM relationship. In the single versus multiple BDM relationship, the LSD_{i} of the preceding activity I is determined by the minimum LSD among the BDM relationships of activities I, J1, J2, and J3. Equation (3) expresses a formula to determine the LSD_{i} of the preceding activity I through the backward pass computation in the single versus multiple BDM relationship.

\[ \text{LSD}_{i} = \min_{j} \left( \text{LSD}_{j} + d_{j} - d_{i} \right) \]

The symbol \( \min_{j} \) in equation (3) means that the minimization is to be over all the beelines IJ that burst from activity I. This research verifies equation (3) through the simple example of the single versus multiple BDM relationship. Fig.10. shows the single versus multiple relationship of the BDM wherein activity A bursts into activities B, C, and D. Activities A and B have a "5-2" BDM relationship, activities A and C have a "10-2" BDM relationship, and activities A and D have a "13-3" BDM relationship.

The LSD_{i} of the preceding activity A in the BDM relationships with the succeeding activities B, C, and D is calculated by applying equation (3) as follows: the first LSD_{i} of the preceding activity A from the "5-2" relationship with activity B is calculated as LSD_{a} = 33 + 2 - 5 = 30; the second LSD_{i} from the "10-2" relationship with activity C is determined as LSD_{a} = 34 + 2 - 10 = 26; and the third LSD_{i} from the "13-3" relationship with activity D is computed as LSD_{a} = 38 + 3 - 13 = 28. The minimum value of "26" then is selected as the LSD_{i} of the preceding activity A and the LFD_{i} is calculated as LFD_{a} = 26 + 15 = 41 by applying equation (4).

From the above, the backward pass computation of the BDM relationship proposed in this research is verified as simple, obvious, and reasonable, as was the forward pass computation.

3.3.3 Computation of Free Float in the BDM

The free float (FF) is defined as the time span within which the completion of an activity may occur without delaying either the completion of the project or the start of any following activity (Harris 1978). During the forward pass computation, a difference between the early start date of an activity and the early finish date of the preceding activity may occur; this is called a link lag (Harris 1978). The link lag (LAG_{ij}) between the preceding activity I and the succeeding activity J in the PDM is defined as equation (5).

\[ \text{LAG}_{ij} = \text{ESD}_{j} - \text{EFD}_{i} \]

A link lag (LAG_{ij}) in the BDM can be defined as a difference between the connecting points of two successive activities; thus, it is stated as equation (6).

\[ \text{LAG}_{ij} = (\text{ESD}_{j} + d_{j}) - (\text{ESD}_{i} + d_{i}) \]

When a link lag occurs in the BDM network, a beeline is modified into an offset-screwdriver shape with a horizontal line that matches the extent of the link lag’s duration, as shown in Fig.11. This unique representational method of a link lag in the BDM allows the time span between the early finish of the predecessor and the early start of the successor to be visually recognizable, something that is impossible in the PDM.

The free float also can be defined as the minimum value of the link lags (Harris 1978). If the single preceding activity I is connected to the multiple succeeding activity Js, the free float of activity I (FF_{i}) is the minimization of the LAG_{ij} and can be expressed as equation (7).

\[ \text{FF}_{i} = \text{LAG}_{ij} = \min_{j} (\text{ESD}_{j} + d_{j}) - (\text{ESD}_{i} + d_{i}) \]

The symbol \( \min_{j} \) in equation (7) means that the minimization is to be over all the beelines IJ that begin with activity I. Fig.12. shows an example for computing the free float of activities in the BDM by applying equations (6) and (7).

In Fig.12., activities A and C have a beeline relationship of "4-2", activities A and D have a beeline relationship of "6-2", and activities B and D have a beeline relationship of "7-3". If the early start dates of activities A, B, and C already have been derived, then, through the application of equation (6), the LAG_{AC} a link lag between activities A and C, is calculated as LAG_{AC} = (15 + 2) - (10 + 4) = 3, the LAG_{AD} a link lag between activities A and D, is found as LAG_{AD} = (16 + 2) - (10 + 6) = 2, and the LAG_{BD} a link lag between
activities B and D is computed as $\text{LAG}_{\text{BD}} = (16 + 3) - (12 + 7) = 0$. Through the application of equation (7), the $\text{FF}_A$, the free float of activity A, is derived as $\text{FF}_A = \text{Min}(\text{LAG}_{AC}, \text{LAG}_{AD}) = \text{Min}(3, 2) = 2$ and the $\text{FF}_B$, the free float of activity B, is derived as $\text{FF}_B = \text{Min}(\text{LAG}_{BD}) = \text{Min}(0) = 0$.

The basic concept for deriving the free float of an activity in the BDM is almost identical with the concept used in the PDM. In the BDM network, however, a free float is calculated based on the beeline connecting points between two consecutive activities.

### 3.3.4 Computation of Total Float in the BDM

The total float (TF) is defined as the time span in which the completion of an activity may occur and not delay the termination of the project (Harris, 1978), and it is the maximum float that an activity could possess. The total float of an activity can be derived from a difference between the forward and backward pass computations. Therefore, $\text{TF}_I$ of activity I can be computed by a difference between $\text{LSD}_I$ and $\text{ESD}_I$, or $\text{LFD}_I$ and $\text{EFD}_I$, as expressed in the equation (8).

$$\text{TF}_I = \text{LSD}_I - \text{ESD}_I = \text{LFD}_I - \text{EFD}_I \quad (8)$$

The concept for deriving the total float of an activity in the BDM is identical with the PDM because the BDM performs the forward and backward pass computations as the PDM does.

### 4. Verification of the BDM

This section verifies the basic concept, principle, and schedule computation methods of the BDM proposed in this research; further, it determines whether or not they are reasonable when they are applied to the complete BDM network for construction projects. The complete BDM network was constructed with 15 activities with various BDM relationships and the schedule computations were performed. The complete BDM network and its schedule computation results, with a critical path of A-C-G-H-L-O, are illustrated in Fig.13.

The results of the complete schedule computations performed in the BDM network confirm that the basic concept and principle of the BDM have been applied reasonably. The BDM thus has all the key elements to evolve into a new networking technique that could replace the existing ADM and PDM.

### 5. CPM Schedule Summarizing Function in the BDM

The BDM technique can represent multiple overlapping relationships between the activities described in the fourth linkage representation type of this paper. This characteristic allows presentation of all schedules within a schedule hierarchy with an identical CPM format, as well as summarizing a lower level's CPM schedule in an upper level's CPM schedule automatically. This summarizing function of the BDM network can be explained with the following example (Kim 2011).

#### 5.1 Work Breakdown Structure and Schedule Hierarchy of Interior Works

Fig.14. shows a work breakdown structure (WBS) of interior works in an apartment unit of a high-rise residential building project. The WBS has three levels. Interior works begin with plastering work after the building structure is completed, and will be completed with cleaning and final inspection. The first level of the WBS is the project itself, the second level shows the major works of the project, and the third level represents the detailed work items to be included in a major work. The WBS generally provides the foundation of the schedule hierarchy; thus, the schedule hierarchy of interior works is composed of three levels that are identical to the WBS.

#### 5.2 Schedule Summarizing Function in the BDM

Fig.15. shows an example of representing a three-level schedule by the BDM. The lowest level or detailed schedule in Fig.15. shows the relationships between work items of WBS level 3. The middle level schedule that is summarized from the lowest level represents the multiple overlapping relationships between the major works of level 2. The highest level or milestone schedule that is summarized from level 2 shows a summarized schedule of the major milestones of the project as one activity, that has multiple milestones. From Fig.15., I can confirm that all BDM schedules within the schedule hierarchy have been summarized from the lower to higher level, maintaining the identical CPM format. Also, the BDM schedule in Fig.15. shows the multiple relationships on intermediate milestones between door and window work and glass work, and furniture and flooring work in the middle schedule; level 2, which is not able to be expressed in the PDM. This unique feature of the BDM shows that it could overcome the limitations and inefficiencies of the existing ADM and PDM.

#### 5.3 Summarized BDM Schedules in the Real Project

A real project that shows the CPM summarizing function in the BDM has almost the same WBS components of an apartment unit interior, as in Fig.14. Figs.16. thru 18. show three real BDM networks
that show the CPM schedule-summarizing function performed by the Beeliner, which is a scheduling software based on the BDM technique. Fig.16. shows the BDM network at level 4, which is the lowest level of the WBS. Fig.17. shows the BDM network at level 3, to be summarized from level 4. Fig.18. shows the BDM network at level 2, to be summarized from level 3.

From these real project examples, I can confirm that the BDM network shows the overall CPM schedule as can the ADM, and also represents the overlapping relationships between activities, such as the PDM. Furthermore, every schedule on all levels of the WBS can be expressed in the identical CPM format, because the BDM can express two or more overlapping relationships between activities.

6. Conclusions

Construction projects are becoming larger and more complex. They require more flexible and innovative scheduling techniques that can be applied in all kinds of project management environments. I have proposed the Beeline Diagramming Method (BDM) as a new networking technique that can represent all the overlapping relationships between activities. This paper defines the basic concepts, principles, interpretation methods, and schedule computation methods that the BDM requires in order to be an effective scheduling technique. To verify the BDM’s adaptability and validity,
the techniques proposed in this study have been applied
to a complete BDM network.

The schedule formats within the schedule hierarchy
of a construction project are generally represented by
different scheduling techniques, because traditional
scheduling techniques, such as ADM or PDM, cannot
represent all the schedules within a schedule hierarchy
in the one format. In contrast, the BDM technique can
represent all the levels within the schedule hierarchy
in CPM format. This paper shows how the multiple
overlapping representations of BDM can provide a
summary of all schedules within the schedule hierarchy,
while maintaining an identical CPM format, and presents an example of representing all the levels within a schedule hierarchy by the BDM technique, as well as the BDM networks of levels 2, 3, and 4 in a construction project. I expect that the BDM as a new networking technique, and its summarizing function, will help project management teams prepare and control project schedules more efficiently and realistically.

References