

Probability

Probability is central to project management. We need to understand the probability of achieving cost or time outcomes and the probability of a risk event occurring. Probability is also closely aligned with possibility and creativity. Certainty eliminates the need to consider options. Only if you consider there is a possibility the current situation / solution is not optimal can you investigate possible improvements.





































The problem with probability is understanding what we know and what we don't – society is built on certainty. Certainly Western culture for the last 2400 years has built on the thinking of Socrates, Plato and Aristotle that uses logic and argument to determine the truth. These ideas were central to the Churches of the Middle Ages (it's very difficult to be a martyr for a probability) and is a core doctrine of our judicial systems and modern management. In Western society, argument based on logic defines the truth, but this is rarely a single option!

Probability theory

Based on this very successful paradigm, modern risk management practices have developed analytical methodologies to determine the probability of events occurring (or not occurring) that allows contingencies to be calculated based on mathematical certainties. At a simple level is a perfectly sensible process. The probability of a 'six' showing if you role one dice once is 1/6 (there are 6 sides and the answer will be 1, 2, 3, 4, 5, or 6). However, if you roll a pair of dice, the probability of one 'six' showing is 11/36 (not 2/6 or 1/3).

Probability is not an addition process – whilst there is exactly 1/6 probability of rolling a 6 if you role a dice once, there is not a 6/6 or 100% guarantee that if you roll the same dice 6 times you will have rolled a six. The probability is fairly high that there would be at least one six, but you may roll more than one and you may not have rolled any.

If we look at all of the possibilities from the rolling of a pair of dice in the diagram below there are 36 options. On six occasions the red dice shows a 'six', and on six occasions the white dice shows a 'six' but one of these options is when the dice show a 'double six'

The calculation is:

- The probability of the first dice showing a 'six' is 1/6
- The probability of the second dice showing a 'six' is 1/6
- And the probability of both dice showing 'six' is 1/36

Therefore the probability of only 1 six showing is: $5/36 + 5/36 = 10/36^1$ And the probability of at least one 'six' (ie, either 1 or 2 sixes), the answer would be: $6/36 + 6/36 - 1/36 = 11$ the 'double six' option would not matter but cannot be counted twice.

The way this is calculated (in preference to using the graphic) is to take the number of ways a single die will **not** show a 6 when rolled (five) and multiply this by the number of ways the second die will **not** show a 6 when rolled. (Also five.) $5 \times 5 = 25$. Subtract this from the total number of ways two dice can appear (36) and we have our answer...eleven. (source: <http://www.edcollins.com/backgammon/diceprob.htm>)

The basis of most probability calculations are similar to the above and rely on two factors. The first is the 'classification' of the event; the second is the assumption there is a bounded range of outcomes. However, even in these circumstances, there is a significant difference between the 'chance' of an event occurring and the 'probability' of it occurring.

Every one knows the chance of a coin when it is tossed landing on 'heads' is 50%; the coin will either land on 'heads' or 'tails'. The first vital consideration is that each throw of the coin is independent. Therefore for any single toss of the coin there is always a 50% chance of 'heads' being the outcome even if the coin has landed on 'heads' in 10 or even 100 previous throws. However, whilst the 'odds' in favour of any single toss landing on 'heads' is 1 to 1 (or evens) the probability of 10 consecutive throws landing on 'heads' is extremely low.

The probability of 'heads' being tossed 10 times in a row is $1/2$ to the power of 10 = 0.00048828125 (or roughly 1/2000). But remember whilst the probability of tossing 10 'heads' in a row is very low, the chance of any single toss of the coin coming up 'heads' remains 50/50.

Similarly, whilst over many thousands of rolls of the pair of dice, the average number of times a single six will appear will be close to 10/36 and the chance of any one roll of the dice showing a single six is 10/36; the probability of seeing 10 single 'sixes' from just 36 rolls of the dice is extremely low.

People are not very good at understanding probability. Most would feel that the difference between a 98% success rate and a 99% success rate would be very small. However, if the tests are being undertaken regularly and a test has a 99% chance of being successful, the probability of 50 successful tests in a row is 61.73% this is calculated by multiplying $0.99 * 0.99$ fifty times. If the probability of a successful outcome is 98% the probability of 50 successful tests in a row reduces to 37.92%.

Another example:

What is the expected outcome from a game where you have a 60% chance of doubling your bet and a 40% chance of losing your bet? The answer if the amount gambled is \$1, is a \$0.20 gain on your \$1.00 bet.

40% of the time you get back nothing - ie. you lose your \$1

60% of the time you get back \$2 - ie. you gain a \$1 over your \$1 bet

Therefore, if you placed 100 bets? In a perfect world

For 40 of the bets you get back nothing. (40x\$0)

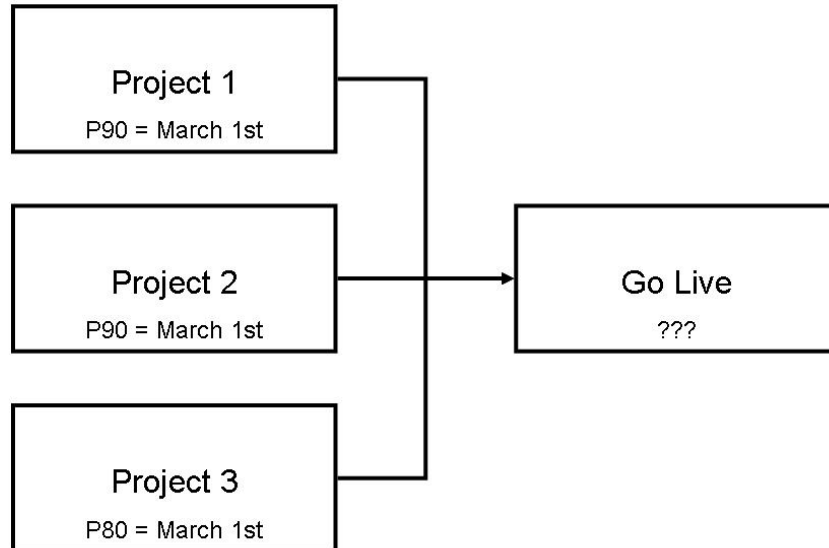
For 60 of the bets? You'd get back \$120. (60x\$2)

You've bet \$100, you get back \$120. You're making \$0.20 on each \$1 bet. The problem is the world is not 'perfect'.

¹ This is part of 'Probability Theory': when two events occur the probabilities can be added if they are independent (the roll of the first dice has no effect on the roll of the second) but should be multiplied if they are interdependent. However, the probability that two event will both occur can never be greater than the probability each will occur individually.

Applying this to projects:

What's the Probability of going live on the 1st March if the 'Go Live' date is dependent on three independent projects, two have a 90% probability of achieving the target date and one has an 80% probability?



The solution to this question is simple but complex....

There is a 1 in 10 chance the 'Go Live' date will be delayed by Project 1

There is a 1 in 10 chance the 'Go Live' date will be delayed by Project 2

There is a 2 in 10 chance the 'Go Live' date will be delayed by Project 3

What is the probability of going live on March 1st?

To understand this problem we need to go back to look at the role of dice:

If you role the dice and get a 1 the project is delayed, any other number it is on time or early.

If you role 1 dice, the probability is 1 in 6 it will land on 1 = 0.1666 or 16.66%

therefore there is a $100 - 16.66 = 83.34\%$ probability of success.

Similarly, if you roll 2 dice, there are 36 possible combinations, and the possibilities of losing are: 1:1, 1:2, 1:3, 1:4, 1:5, 1:6, 6:1, 5:1, 4:1, 3:1, 2:1. (11 possibilities – in this situation there will be a delay if one project is late and there will still e a delay if both projects are late)

The way this is calculated (as discussed above) is to take the number of ways a single die will **not** show a 1 when rolled (five) and multiply this by the number of ways the second die will **not** show a 1 when rolled. (Also five.) $5 \times 5 = 25$. Subtract this from the total number of ways two dice can appear (36) and we have our answer...eleven.

Therefore the probability of rolling a 1 and being late are $11/36 = 0.3055$ or 30.55%, therefore the probability of success is $100 - 30.55 = 69.45\%$ probability of being on time.

Now if we roll 3 dice we can extend the calculation above as follows:

The number of possible outcomes are $6 \times 6 \times 6 = 216$

The number of ways not to show a 1 are $5 \times 5 \times 5 = 125$

Meaning there are 216 combinations and there are 125 ways of **not** rolling a 1 leaving $216 - 125 = 91$ possibilities of rolling a 1

$91/216 = 0.4213$ or 42.13% probability of failure therefore there is a $100 - 42.13 = 57.87\%$ probability of success.

So going back to the original problem:

- Project 1 has a 1 in 10 chance of causing a delay
- Project 2 has a 1 in 10 chance of causing a delay
- Project 3 has a 1 in 5 chance of causing a delay

There are $10 \times 10 \times 5 = 500$ possible outcomes and within this $9 \times 9 \times 4 = 324$ ways of not being late. $500 - 324$ leaves 176 ways of being late. $176/500 = 0.352$ or a 35.2% probability of not making the 'Go Live' date. Or a $100 - 35.2 = 64.8\%$ probability of being on time.

The quicker way to calculate this is simply to multiply the probabilities together:

$$0.9 \times 0.9 \times 0.8 = 64.8\%$$

What is important to note here is we are looking at a binary outcome – the probability of 'going live' on the due date, the answer is Yes or No! If you want more useful information such as answers to the question 'how late' simple probability calculations are not sufficient.

Why simple numbers are not enough

Here's a short example used by Glen Alleman (http://herdingcats.typepad.com/my_weblog/).

We are going to send you to Trinidad Tobago for one year, with a hat, a beach chair, a clip board and a pencil. I want you to write down the high temperature of the day for 365 days. From this data we are going to determine the "most likely temperature," the temperature that occurs most often.

Now we are going to send you the Cody Wyoming to do the same thing for 365 days. Write down the high temperature of the day for 365 days.

It turns out the "most likely" temperature – the number that occurs most often – in both locations is close to each other at 84 degrees F. but of course the variance between Trinidad and Cody is not the same. In Trinidad the range is between 70 to 90 and in Cody the range between -30 and 103, but the Most Likely number – not the mean, or median – but the mode is 84. Based on this average are you going to take a beach chair and sun hat to Cody in January???? Or is the range information equally if not more important!

Bounded v Unbounded Ranges

The problem with project data is that any probability calculation is based on the assumption of a finite range of variables; there are only 36 possible options for a pair of dice. Conversely, there are no limits to many aspects of project risk. Consider the following:

- You plot the distribution and average the weight of 1000 adult males. Adding another person, even if he is the heaviest person in the world only makes a small difference to the average. No one weighs a ton! The results are normal (Gaussian-Poisson) and theorems in probability such as the Law of Large Numbers and Least Squares (Standard Deviation) apply.
- You plot the distribution and average the net wealth of 1000 people. Adding Bill Gates to the group causes a quantum change in the values. Unlike weight, wealth can be unlimited. Gaussian-Poisson theories do not apply!



Most texts and discussion on risk assume reasonable/predictable limits. Managing variables with no known range of results is rarely discussed and many project variables are in this category².

Classification of Events

The next problem with project data is finding ‘similar data’. Risk management tools and methodologies such as Monte Carlo assume probabilities (or more correctly, probability distributions) for durations, costs, etc to be either known, or they can be accurately determined from relevant historical data. The word relevant is critical: it emphasises that the data used to calculate probabilities (or distributions) should be from situations that are similar to the one at hand. *This apparently simple requirement papers over a fundamental problem in the foundations of probability: the reference class problem*³.

What does ‘similar’ mean? Do we look at projects with similar scope, or do we use size (in terms of budget, resources or other measure), or technology or some other measure? There could be a range of criteria that could be used, but one never knows with certainty which are relevant in the particular circumstances. This is an issue because the probability changes depending on the classification criteria used..... this is the reference class problem.

Creating a probability distribution based on data for completed projects that are similar to the one of interest requires:

- Collecting data for a number of similar past projects – these projects form the reference class. The reference class must encompass a sufficient number of projects to produce a meaningful statistical distribution, but individual projects must be similar to the project of interest.
- Establishing a probability distribution based on reliable data for the reference class. The challenge here is to get good data for a sufficient number of reference class projects.
- Predicting most likely outcomes for the project of interest based on comparisons with the reference class distribution.

Simple until you realise by definition we never do the same project twice; each project is a ‘*temporary endeavour, having a defined beginning and end, undertaken to meet unique goals and objectives*’. At best you can gather data from ‘similar’ projects selected on the basis of common sense or expert judgement or some other subjective approach.

The reference class problem affects most probabilistic methods in project management⁴. It is a problem because it is often impossible to know, which attributes of the projects are the key variables that relate to the event of interest. Consequently it is impossible to determine with certainty whether or not a particular event or project belongs to a defined reference class.

The impact of probability on the management of projects

The probability of each project being either successful or failing is 1 (ie, it will either succeed or fail). If management decisions shift the odds on succeeding to an 80% probability of success, the probability of failing drops to 20%: $0.8 + 0.2 = 1$. As management invests in shifting this ratio towards 100% success the

² For more on probability and risk see:

The Meaning of Risk in an Uncertain World: http://www.mosaicprojects.com.au/Resources_Papers_040.html

Scheduling in the Age of Complexity: http://www.mosaicprojects.com.au/Resources_Papers_089.html

³ Quoting from Eight to Late, *The reference class problem and its implications for project management:*

<http://eight2late.wordpress.com/2010/05/13/the-reference-class-problem-and-its-implications-for-project-management/>

⁴ Bayesian Networks offer one solution to this problem but are very complex to establish for short term one-off situations such as a project.



probability of failure reduces, but the costs increase exponentially. Then even having done this, there is still a probability of failure and the money wasted in buying the excessive 'safety factor' cannot be used for other value creating activities; a balance is needed⁵.

This 'balance is often assessed using techniques such as Monte Carlo analysis of a schedule, a cost plan or both. But this is always subjective and needs skilled application.

- Cost is relatively straightforward and is one of the few situations where the 'flaw of averages'⁶ does not apply. Each cost item is discrete and therefore the outcome from analysing each individual item will deliver a similar result to an analysis of a set of summary costings.
- Schedule activities are interdependent and present a unique set of challenges. A long string of variables (ie a chain of activities with range assessments) is subject to the effect of the Central Limit Theorem (CLT) where the 'swings and roundabouts' tend to cancel each other out and the overall range outcome can be understood (but applying some form of correlation to off set this can be risky as well). Simplify the network by using summary activities and you remove the effect of 'merge bias' discussed in the multi-project example on page 8 of this paper (the effect on merging paths in a network is the same and summary networks have fewer merge points than detailed networks). However, ignoring variability and using a CPM scheduling based on most likely durations (the 'mode') and you are simply creating an optimistic analysis based on the strong version of flaw of averages⁷. Getting the balance right needs expertise⁸.

When management ignores the possibility of failure they see each project failure as a unique surprise and then look for someone to blame; this is despite the fact every single project has an intrinsic probability of failing! However, if management accepts there is a probability of failure in every project all of the time they can take prudent steps to monitor for emerging issues⁹. Probability is not certainty and prudent reactions at the right time can change future outcomes.

⁵ For more on the balance between the probability of failing and the cost of safety see, **Stakeholder Risk Tolerance:** <http://mosaicprojects.wordpress.com/2012/04/03/stakeholder-risk-tolerance/>

⁶ For more on the '**flaw of averages**' see: <http://mosaicprojects.wordpress.com/2012/06/22/the-flaw-of-averages/>

⁷ For more on **why CPM is wildly optimistic** see: http://www.mosaicprojects.com.au/Resources_Papers_117.html

⁸ The mathematics of probability basically say you cannot combine two known probability ranges to calculate the outcome of the combination. This is the key issue that causes a problem know as PERT Merge Bias*. If everything is in sequence (A follows B follows C) and everything is independent then you can get to an answer but the calculations are very complex - the PERT solution was very simplistic (a fact known to the mathematicians that developed PERT).

What anyone approaching this type of problem in the modern world does is first determine the probable range of outcomes for each activity using a three point estimate (maximum, minimum, most likely) then determine the probable distribution of outcomes within that range (Triangular, Beta, Normal, etc.), then run their model through a Monte Carlo simulation. Monte Carlo simply calculates the model 100s of times using different, random, values from within the ranges each time (but tracking each of the individual distribution profiles). The tool then plots the of outcomes of all of the 'runs' showing how often any given value was achieved. From this plot you can work out the probability of any cost or duration being achieved. Monte Carlo deals with all of the issues such as most outcomes for any one of the events are more likely to be nearer to the most likely than at an extreme outlier; but occasionally you will get very high (or very low) outcomes across all of the items being considered. In project management Triangular or Beta distributions are typically used because there is a lot more things that can go wrong than are likely to go right (Normal distribution assumes an equal probability of plusses or minuses).

The number of iterations of the calculation needed to generate a reliable outcome are dependent on when 'stability' is reached - this means doing another 200 or 300 runs makes no practical difference to the results from the model. Typically for smaller models stability is achieved with 500 runs larger ones typically use 1000 but these are arbitrary numbers and both designed to be well above the number really needed - but all that's involved is a few more seconds of processing time..... Its brute force over science but Monte Carlo** is the only thing that works.

* See page 8 of: http://www.mosaicprojects.com.au/WhitePapers/WP1087_PERT.pdf

** For more on Monte Carlo see http://www.mosaicprojects.com.au/Mag_Articles/P006_Predicting_the_Future.pdf

⁹ For more on effective **project surveillance** see: http://www.mosaicprojects.com.au/WhitePapers/WP1080_Project_Reviews.pdf



Conclusion

There are no simple solutions to this problem, but hopefully this paper will encourage you to seek expert advice. Basing project decisions on the assumption the data being used is accurate is doomed to failure – it is impossible to predict the future with certainty (if we could casinos and bookmakers would go bankrupt) and then there are the unknown unknowns we simply know nothing about¹⁰. We know enough to know some projects will probably fail but that is all we can know for certain. We do not have the capability of predicting which project will fail, of the precise probability of any project failing, or the overall probability of success and failure in the whole portfolio beyond general assumptions. However, the purpose of analysis is not to provide a ‘correct’ answer! No amount of analysis can ever provide certainty, no matter how much analysis is applied, what good analysis does is provide insight to help shift the balance of probability into your favour.

Risk assessments, Monte Carlo analysis and other techniques will provide valuable insights if they are used wisely. What they cannot do, particularly in a ‘one-off’ event such as a project is provide certainty. Unlike motor vehicle and life insurance companies (where the Law of Large Numbers apply), in project management, we simply don’t have enough similar data for probabilistic analysis to provide accurate assessments of the probable range of outcomes.

Skilful project managers and their executive management recognise this lack of certainty, do sufficient analysis to develop a reasonable level of insight (avoiding analysis paralysis) and then manage the ongoing work knowing their estimates are not correct. This allows the key question of ‘how wrong are the estimates’ to be asked and effective surveillance systems established to identify emerging problems and opportunities.

Knowing your estimates are wrong also allows system to be put in place so that as the expected variances start to emerge, routine adjustments to the project plans are made to lock in gains and mitigate losses. Adapting to an uncertain future requires different skills to those developed by the ‘scientific management school’ of the 19th Century but are essential in the ‘age of complexity’¹¹.

Risk White Papers

Mosaic’s risk White Papers are:

- Risk Management: http://www.mosaicprojects.com.au/WhitePapers/WP1047_Risk_Management.pdf
- Types of Risk: http://www.mosaicprojects.com.au/WhitePapers/WP1057_Types_of_Risk.pdf
- Risk Assessment: http://www.mosaicprojects.com.au/WhitePapers/WP1015_Risk_Assessment.pdf
- Probability: http://www.mosaicprojects.com.au/WhitePapers/WP1037_Probability.pdf
- Our blog posts on risk are at: <http://mosaicprojects.wordpress.com/category/project-controls/risk/>

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This White Paper is part of Mosaic’s **Project Knowledge Index** to view and download a wide range of published papers and articles see: http://www.mosaicprojects.com.au/PM-Knowledge_Index.html

¹⁰ For more on **unknown unknowns** see: http://www.mosaicprojects.com.au/WhitePapers/WP1057_Types_of_Risk.pdf

¹¹ See **Scheduling in the Age of Complexity**: http://www.mosaicprojects.com.au/Resources_Papers_089.html

