Defining the Critical Path

One of the more perplexing issues facing the scheduling profession is defining precisely what constitutes the ‘critical path’ within a CPM schedule. There are many different descriptions in regular use some are:

- Longest sequence of activities in a project plan which must be completed on time for the project to complete on due date. An activity on the critical path cannot be started until its predecessor activity is complete; if it is delayed for a day, the entire project will be delayed for a day unless the activity following the delayed activity is completed a day earlier.

- The critical path is defined as the longest path through the CPM network and consists of those activities that determine the shortest time for project completion. Activities within this or list form a series (or sequence) of logically connected activities that is called the critical path. A delay to the start or completion of any activity in this critical path results in a delay to project completion, assuming that this path consists of a continuous sequence of activities without an overriding date constraint or multiple calendars. [AACEi - RP 49R-06]

- The series of tasks that must finish on time for the entire project to finish on schedule. Each task on the critical path is a critical task.

- The sequence of activities that represents the longest path through a project, which determines the shortest possible duration. [PMBOK® Guide 5th Edition]

- In a network diagram, the longest path from start to finish or the path without any slack, and thus the path corresponding to the shortest time in which the project can be completed.

- In a project network diagram, the path with the longest duration. The critical path may change from time to time as activities are completed ahead of or behind schedule. See critical path method.

- The line of project activities having the least float, especially when float is close to, or below zero.

- Generally, but not always, the sequence of schedule activities that determines the duration of the project. Generally, it is the longest path through the project (or from the current data date). However a critical path can end…. in the middle of the project schedule. [The Practice Standard for Scheduling 2nd Edition – PMI]

- The route through the network that has only critical activities.

- The series of consecutive activities that represent the longest path through the project.

- The series of interdependent activities of a project connected end-to-end, which determines the shortest total length of the project. The critical path of a project may change from time to time as activities are completed ahead of or behind schedule.

- The series of activities that define the total project duration.

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1 The term ‘critical path’ was devised by the PERT project in 1957. There were a number of parallel projects developing CPM type programs in both the UK and the USA with various names for the ‘main chain’ of activities. By 1959 everyone seemed to have settled on the name ‘critical path’. For more on the history of CPM see: https://mosaicprojects.com.au/PDF_Papers/P042_History_of_Scheduling.pdf

2 CPM = Critical Path Method

3 Many definitions sourced from: http://www.maxwideman.com/pmglossary/PMG_C15.htm
The path (sequence) of activities which represent the longest total time required to complete the project. A delay in any activity in the critical path causes a delay in the completion of the project. There may be more than one critical path depending on durations and work flow logic.

A sequence of activities through a project network from start to finish, the sum of whose durations determines the overall project duration. [APM BoK 5th Edition]

The sequence of activities that must be completed on schedule for the entire project to be completed on schedule. It is the longest duration path through the work plan. If an activity on the critical path is delayed by one day, the entire project will be delayed by one day (unless another activity on the critical path can be accelerated by one day).

Many of these descriptions are discussions rather than a definition and several are circular or contradictory.

Why the Critical Path needs definition

![Critical Path Diagram]

Having an unambiguous definition of the Critical path is important because many contracts will only provide an extension of time for excusable delays if the delay impacts a ‘critical activity’ or causes an activity to become critical. At a more basic level, understanding what is important from the viewpoint of criticality is central to the whole philosophy behind the ‘critical path method’ of scheduling.

The definition of ‘critical activity’ is generally agreed as being those activities that make up the ‘critical path’. However, as can be seen from the above, what is not generally agreed is the definition of what constitutes the ‘critical path’.

Whilst creating a precise and unambiguous definition of the ‘critical path’ is important; it is far from simple primarily due to advances in scheduling practice that allow the inclusion of both date based and other forms of constraint at any point in a schedule.

What is an ‘Activity Path’

Project Management literature has largely failed to define the term path or activity path in relation to CPM scheduling. However, before we can place the adjective ‘critical’ in front of the noun ‘path’ and derive a meaningful definition for the combined term, the object noun itself needs to be defined without creating a circular definition. Any definition that uses a characteristic of the path to define the path eg, Total Float, Longest, Driving, etc., or a derived characteristic such as critical, near-critical, non-critical, primary, secondary, etc., is circular, and therefore inconclusive.

The definition proposed by Murray Woolf in his book, CPM Mechanics offers an answer:

1) An Activity Path is a chain of activities that are connected one to another in a logical sequence that span from the Path Start Activity to the Path End Activity.

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4 The various type of constrain are discussed in Basic CPM Calculations, page 12:
5 These concepts are more fully defined in Murray Woolf's book CPM Mechanics.
2) **Path Start Activity**: The first activity (or Milestone) in an Activity Path, and is one which either has no predecessor or which is constrained by a legitimate* Start Date Constraint.

3) **Path End activity**: The last activity (or Milestone) in an Activity Path, and is one which either has no successor or which is constrained by a legitimate* Finish Date Constraint.

*Where a ‘legitimate date constraint’ is one that represents either an express management intention, or a contractual or other obligation or limitation affecting the scheduling of the work.

The consequence of this definition is to identify at least one distinct Activity Path between each pair of Path Start Activities and the Path End Activities that are logically connected to each other. If Deadline X can be traced back to Start Milestones A, B, and C, then there are at least three Activity Paths feeding into Deadline X. Having defined a path, we can now consider which, if any of the paths are ‘critical’.

**Critical Path -v- Critical work!**

A number of authors including Murray Woolf⁶ have criticised the concept of ‘critical path’ preferring to focus on critical work. The concept of ‘critical’ can be extended to include:

- Near-critical path activities with low total float (say, less than 5 days) that may become the controlling path of activities if the low total float is used-up.
- Hyper-critical with negative float. This means, the activity is already delaying a constrained date.
- Activities in-progress or due to start because of their affect on resource utilisation, efficiency and costs.
- High-cost, long duration lead time, or other very important activities

The effect of resource utilisation also has a profound effect on the concept of the ‘critical path’; this is discussed in our paper “Resource optimisation - a new paradigm for project scheduling”⁷.

Shift from the deterministic single estimates used in CPM to probabilistic (three point, etc.) estimates used in Monte Carlo and the critical path moves around, two standard reports are tornado charts showing how frequently an activity is on the ‘critical path’ and one showing the correlation between changes in an activity’s duration and changes in the overall project duration.

In summary, not all critical activities are on the critical path and not all of the activities on the critical path need to be considered critically ‘important’ activities. Common sense is needed to interpret the schedule information sensibly!

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6 See Faster Construction Projects with CPM by Murray Woolf.
**Multiple Critical Paths**

Whilst it is probably not ‘good practice’, there may be multiple critical paths through a schedule these may occur for a number of reasons including:

- Different paths having exactly the same overall duration.
- Constrained milestones reflecting phased handovers / completions / contractual requirements may cause different paths to be critical at the same time.
- Individual schedules, each with their own critical path, combined into an overall master program.

If there is more than one critical path, it is important to understand the reason.

**Why Float never defines the Critical Path**

Float is never the definition of the critical path. Float is a consequence of calculations\(^8\) as these two examples show:

- **Calendars:** The engineering work on your project is scheduled to work on a 24 x 7 calendar. The commissioning work requires client staff in attendance and is scheduled on an 8 to 5, 5 day week calendar. The last engineering task on the critical path, needed to allow commissioning to start finishes at 21:00 (9:00PM) on Friday, its immediate successor on the critical path, with Zero days float is scheduled to start at 8:00 on Monday. The engineering activity has 59 hours float (2 days if the time unit is days) and is on the critical path.

  The project team focus on the ‘critical activities’ and accelerate the work by 6 hours. The Engineering activity now finishes at 15:00 (3:00pm) and the commissioning work can start immediately - the engineering activity float changes to Zero - same activities, same logic but the ‘float values’ are changed by effect of the activity calendars and precisely when the work occurs.

- **Constraints:** Your major project has an official opening involving international dignitaries scheduled for the 30\(^{th}\) August. The opening and subsequent clean up and handover are on the critical path. However, the last of the construction work prior to the opening is scheduled to finish on the 25\(^{th}\) August – the path prior to the opening has 5 days float (depending on calendars and weekends). After the opening the last part of the path has Zero float (assuming no other date constraints).

  The float values change based on constraints and calendars and where the activities start and finish. In scenario 1, when the engineering works completed 6 hours earlier, the commissioning could start on Friday afternoon and the Float reduced to Zero. In scenario 2 the longest path from the schedule start to the opening has 5 days float, but still represents the most time critical path.

**A New Definition of the Critical Path**

Most authorities seem to have agreed the following basic points:

- The use of the term ‘critical’ in project scheduling has a precise meaning that is different from the general dictionary meaning.
  In scheduling, ‘critical activities’ make up the ‘critical path’. Other activities may be critically important in the normal dictionary meaning of the term, but they are not ‘critical activities’.

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• The ‘critical path’ does not have to start at the beginning or finish at the end of the schedule, interim constraints, staged handovers, phasing and contractual obligations can affect the start point, the end point and the number of critical paths.

• Float is not a key determinant of ‘criticality’ and the ‘critical path’. If the natural time required to complete the schedule (or a part of the schedule) is less than the contract period, and a date constraint is applied to the final milestone it is quite possible to see a ‘critical path’ with positive float. Conversely, if the natural time required to complete a schedule is more than the contract period, and a date constraint is applied to the final milestone it is quite possible to see a ‘critical path’ with negative float.

• The ‘critical path’ is the path that defines the end point of the schedule (or a part of the schedule). Delays on the ‘critical path’ will delay a project completion; and accelerating work on the ‘critical path’ will advance the project completion (at least until another path becomes critical).9

Based on these foundations, the CIOB10 has developed the following description:

Critical Path = the longest sequence of activities from commencement to completion of a key date, section, or completion of the works as a whole. In relation to each, it is that sequence of activities, which will take the longest to complete or, put another way, the sequence of activities, which will determine the earliest possible finish date. Hence, it is timely commencement and completion of those activities on that path, which will secure completion of the key date, section, or the works as a whole on time.

The above description can be reduced to the following definition:

Critical Path: The sequence of activities that determine the earliest possible completion of the project, a phase of the project, or a sequence of activities that cause an imposed completion requirement to be exceeded.

This new definition11 does not preclude the possibility of several ‘completions’ within the one project to account for interim handovers required under a contract. It allows for the possibility of the critical path starting at the beginning of the schedule or at some interim point where an external dependency allows the ‘critical’ work to start. Additionally, the sequence of activities may be determined logically (through links or dependencies) or through the sequential movement of resources. The definition is both concise and unambiguous.

This definition has been included in ISO 21500 Guide to Project Management, as:

Critical Path: sequence of activities that determine the earliest possible completion date for the project or phase.

Hopefully the world will gradually move to using this definition.

9 The consequences of changing the duration of an activity on the critical path is not always obvious. A number of constructs involving Start-to-Start and/or Finish-to-finish links can have unintended consequences. These logical problems are explained at: https://mosaicprojects.wordpress.com/2016/06/12/critical-confusion-when-activities-on-the-critical-path-dont-compute/


11 In the early versions of the PMBOK® Guide, the definition used was very similar: In a project network diagram, the series of activities which determines the earliest completion of the project. (source Bill Duncan)